

N=2 Super-Born-Infeld Theory Revisited ¹

Sergei V. Ketov ²

*Department of Physics
University of Maryland
College Park, MD 20742, USA*

and

*Institut für Theoretische Physik
Universität Hannover, Appelstr. 2
Hannover 30167, Germany*

ketov@itp.uni-hannover.de

Abstract

I discuss the symmetry structure of the N=2 supersymmetric extension of the Born-Infeld action in four dimensions, and confirm its interpretation as the Goldstone-Maxwell action associated with partial breaking of N=4 extended supersymmetry down to N=2, by demonstrating hidden invariance of the action with respect to non-linearly realized (spontaneously broken) symmetries, in the remarkably simple way. I also argue about the uniqueness of supersymmetric extensions of the Born-Infeld action, and their possible relation to noncommutative geometry.

¹Supported in part by NSF grant # PHY-98-02551

²On leave from: High Current Electronics Institute of the Russian Academy of Sciences, Siberian Branch, Akademichesky 4, Tomsk 634055, Russia

1 Introduction

In ref. [1] I proposed the N=2 supersymmetric extension of the four-dimensional *Born-Infeld* (BI) action. I interpreted it as the Goldstone-Maxwell action associated with spontaneous (partial) breaking of (rigid) N=4 supersymmetry down to N=2, and the N=2 (abelian) vector supermultiplet of Goldstone fields. The basic idea behind this interpretation was the anticipated equivalence (modulo a non-linear field redefinition) between the N=2 super-BI action in four dimensions and the gauge-fixed worldvolume action of a D3-brane propagating in six dimensions. This equivalence was verified in ref. [1], in the leading and subleading orders only (see ref. [2] too), while no direct argument was presented. In this Letter I give the transformation laws of the hidden non-linearly realized symmetries (including spontaneously broken translations and extra N=2 supersymmetry) that unambiguously determine the form of the N=2 super-BI action and prove its Goldstone nature. The related issue of uniqueness of an N=2 superextension of the BI action is discussed too. I also propose an N=2 superconformal extension of the BI theory, and speculate about its possible relation to noncommutative geometry.

2 On fundamental features of bosonic BI theory

In this introductory section I recall some well-known facts about the bosonic BI action, in order to provide a basis for the subsequent discussion of its N=2 supersymmetric extension in sect. 3.

The bosonic BI action in flat four-dimensional spacetime with Minkowski metric $\eta_{\mu\nu}$, $\mu, \nu = 0, 1, 2, 3$, reads ³

$$S_{\text{BI}} = -\frac{1}{b^2} \int d^4x \sqrt{-\det(\eta_{\mu\nu} + bF_{\mu\nu})} \quad , \quad (1)$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, and $b > 0$ is the dimensionful parameter. For instance, in string theory one has $b = 2\pi\alpha'$, whereas in N=1 supersymmetric QED one has $b = e^2/(2\sqrt{6}\pi m^2)$. In what follows, I choose $b = 1$ for simplicity.

The BI theory (1) can be thought of as the particular covariant deformation of Maxwell electrodynamics by higher order terms depending upon F only. In fact,

³The overall normalization of the BI action yields the Maxwell term, $-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$, as the leading contribution. The D3-brane action has, in addition, the inverse string coupling constant in front of the action.

the BI theory also shares with the Maxwell theory some other physical properties, such as causal propagation, positive energy density and electric-magnetic duality (see, e.g., refs. [1, 2] and references therein). Unlike the Maxwell theory, its BI generalization gives rise to the celebrated taming of the Coulomb self-energy, i.e. it smears the singularity associated with a point-like charge in classical electrodynamics. Supersymmetry is known to be compatible with causality, positive energy and duality, so that one expects from supersymmetric BI actions the similar (properly generalized) properties. It is indeed the case for the N=1 BI action [3], and it should be the case for the N=2 BI action [1] too.

As is also quite clear from its origin, either in open string theory or in N=1 scalar QED, the BI action is the *effective* action obtained by summing up certain quantum corrections (to all orders in b) that are independent upon spacetime derivatives (∂F) of the Maxwell field strength F . The effective action is dictated by S-matrix, being defined modulo local field redefinitions. This does not, however, make the BI action to be ambiguous since it depends upon the vector gauge potential A only via its field strength F , while any local reparametrization of A merely results in the additive ∂F -dependent terms that are to be disregarded altogether, by definition of the BI action,

$$\delta S = \int d^4x \delta \mathcal{L}(F) = \int d^4x \frac{\partial \mathcal{L}}{\partial F_{\mu\nu}} \partial_\mu \delta A_\nu = - \int d^4x \partial_\mu \frac{\partial \mathcal{L}}{\partial F_{\mu\nu}} \delta A_\nu = O(\partial F) . \quad (2)$$

In other words, the BI action is the effective action of slowly varying (but not necessarily small) abelian gauge fields, which is dependent upon F , being independent upon ∂F . In supersymmetric BI theories the rôle of F is played by the gauge superfield strength W , so that the super-BI actions in superspace are defined modulo spacetime derivatives of W .

3 N=2 BI action and its symmetries

The Goldstone-Maxwell nature of the N=1 BI action, as the Goldstone action associated with spontaneous partial supersymmetry breaking $N = 2 \rightarrow N = 1$, and the N=1 vector supermultiplet of Goldstone fields, is well-established [3, 2], so that I skip any discussion of it. Both N=1 and N=2 gauge field theories are most naturally formulated in superspace, with manifest off-shell N=1 or N=2 supersymmetry, respectively, which makes a study of partial breaking $N = 2 \rightarrow N = 1$ to be rather straightforward, by starting from a linear off-shell realization of N=2 supersymmetry and imposing a non-linear constraint. Partial breaking $N = 4 \rightarrow N = 2$ in N=2

superspace is more complicated since a natural (off-shell and N=4 supersymmetric) formulation of N=4 gauge theories does not exist.

The N=2 supersymmetric BI action can be formulated in the standard N=2 superspace parametrized by $Z = (x^{\alpha\dot{\alpha}}, \theta_i^\alpha, \bar{\theta}_{\dot{\alpha}}^i)$, where $\alpha = 1, 2$ and $i = 1, 2$. The N=2 flat superspace covariant derivatives $(\partial_{\alpha\dot{\alpha}}, D_\alpha^i, \bar{D}_{\dot{\alpha}}^i)$ satisfy the algebra

$$\{D_\alpha^i, \bar{D}_{\dot{\alpha}j}\} = -2i\delta_j^i \partial_{\alpha\dot{\alpha}}, \quad \{D_\alpha^i, D_\beta^j\} = \{\bar{D}_{\dot{\alpha}}^i, \bar{D}_{\dot{\beta}}^j\} = 0. \quad (3)$$

The standard realization is given by

$$D_\alpha^i = \frac{\partial}{\partial \theta_i^\alpha} + i\bar{\theta}_{\dot{\alpha}}^i \partial_{\alpha\dot{\alpha}}, \quad \bar{D}_{\dot{\alpha}i} = -\frac{\partial}{\partial \bar{\theta}^{\dot{\alpha}i}} - i\theta_i^\alpha \partial_{\alpha\dot{\alpha}}. \quad (4)$$

The $SL(2, \mathbf{C})$ and $SU(2)$ indices are raised and lowered by the use of Levi-Civita (ε) symbols, as usual. I use the notation [4]

$$D^{ij} = \frac{1}{2}D^{\alpha i}D_\alpha^j, \quad D_{\alpha\beta} = \frac{1}{2}D_{\alpha i}D_\beta^i, \quad (D^3)_i^\alpha = \frac{\partial}{\partial D_\alpha^i}D^4, \quad D^4 = \prod_{\alpha,i} D_\alpha^i, \quad (5)$$

and similarly for \bar{D} 's and θ 's ($\bar{\theta}$'s). The standard (Berezin) integration rules imply

$$\int d^4x d^8\theta \mathcal{L} \equiv \int d^4x d^4\theta d^4\bar{\theta} \mathcal{L} = \int d^4x d^4\theta \bar{D}^4 \mathcal{L} = \int d^4x d^4\bar{\theta} D^4 \mathcal{L}. \quad (6)$$

The abelian N=2 superfield strength is described by an N=2 restricted chiral superfield W satisfying the off-shell N=2 superspace constraints

$$\bar{D}_{\dot{\alpha}}^i W = 0 \quad \text{and} \quad D^4 W = \square \bar{W}. \quad (7)$$

The second constraint (7) is just the N=2 Bianchi identity that implies $\square(D_{ij}W - \bar{D}_{ij}\bar{W}) = 0$ and, hence, $D_{ij}W = \bar{D}_{ij}\bar{W}$.⁴ A solution to eq. (7) in components reads (in N=2 chiral superspace parametrized by $y^\mu = x^\mu - \frac{i}{2}\theta_i^\alpha \sigma_{\alpha\dot{\alpha}}^{\mu} \bar{\theta}^{\dot{\alpha}i}$ and θ_i^α)

$$W(y, \theta) = a(y) + \theta_i^\alpha \psi_\alpha^i(y) - \frac{1}{2}\theta_i^\alpha (\bar{\tau})^i_j \theta_\alpha^j \cdot \vec{D}(y) - i(\theta^3)^{i\alpha} \partial_{\alpha\beta} \bar{\psi}_i^\beta(y) + \theta^4 \square \bar{a}(y), \quad (8)$$

where I have introduced the complex (physical) scalar a , the chiral (physical) spinor isodoublet ψ_α^i , the real (auxiliary) isotriplet $\vec{D} = \frac{1}{2}(\bar{\tau})^i_j D^j_i$, and the Maxwell field strength $F_{\mu\nu}$ subject to the Bianchi identity

$$\varepsilon^{\mu\nu\lambda\rho} \partial_\nu F_{\lambda\rho} = 0, \quad (9)$$

⁴The fields are assumed to vanish at infinity.

whose solution is just $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$.

The N=2 supersymmetric extension of the BI action, proposed in ref. [1], reads

$$S = \frac{1}{2} \int d^4x d^4\theta W^2 + \frac{1}{8} \int d^4x d^8\theta \mathcal{Y}(K, \bar{K}) W^2 \bar{W}^2, \quad (10)$$

where $K = D^4 W^2$ and $\bar{K} = \bar{D}^4 \bar{W}^2$, and

$$\begin{aligned} \mathcal{Y}(K, \bar{K}) &= \frac{1 - \frac{1}{4}(K + \bar{K}) - \sqrt{(1 - \frac{1}{4}K - \frac{1}{4}\bar{K})^2 - \frac{1}{4}K\bar{K}}}{K\bar{K}} \\ &= 1 + \frac{1}{4}(K + \bar{K}) + O(K^2). \end{aligned} \quad (11)$$

The action (10) can be rewritten to the form

$$S = \frac{1}{4} \int d^4x d^4\theta X + \frac{1}{4} \int d^4x d^4\bar{\theta} \bar{X} + O(\partial W), \quad (12)$$

where the N=2 chiral lagrangian X is the iterative solution to the N=2 non-linear constraint [1]

$$X = \frac{1}{4} X \bar{D}^4 \bar{X} + W^2. \quad (13)$$

The uniqueness of the N=2 BI action was questioned in ref. [5] by presenting a calculation of some terms in the iterative solution to eq. (13), which are absent in the perturbative expansion of the action (10), for example,

$$\int d^4x d^8\theta W^2 \bar{W}^2 \left[(D^4 W^2) \bar{D}^4 (\bar{W}^2 D^4 W^2) + (\bar{D}^4 \bar{W}^2) D^4 (W^2 \bar{D}^4 \bar{W}^2) \right] \quad (14a)$$

versus

$$\int d^4x d^8\theta W^2 \bar{W}^2 \left[(D^4 W^2)^2 \bar{D}^4 \bar{W}^2 + (\bar{D}^4 \bar{W}^2)^2 D^4 W^2 \right]. \quad (14b)$$

However, it is not difficult to verify, by the use of eqs. (3) and (7), that the difference between eqs. (14a) and (14b) amounts to the ∂W -dependent terms which do not belong to the N=2 BI action because they are ambiguous (*cf.* sect. 2). It was also explicitly demonstrated in ref. [5] that the N=2 BI action (12) is self-dual with respect to an N=2 supersymmetric electric-magnetic duality (claimed in ref. [1] too), by keeping all terms in the solution to eq. (13), including the ∂W -dependent ones. This means that taking into account *some* ∂W -dependent terms is apparently needed to demonstrate the N=2 supersymmetric electric-magnetic duality of the N=2 BI action. In general, however, it does not make sense to keep *some* ∂W (or ∂F) dependent terms in the effective BI action originating either from a quantized open superstring theory or from a quantized supersymmetric gauge theory, while ignoring *other* possible ∂W (or ∂F) dependent quantum corrections. Perhaps, the N=2 electric-magnetic self-duality may, nevertheless, be useful for a study of derivative corrections to the N=2 BI action in a more fundamental framework than just N=2 supersymmetry.

The Goldstone interpretation of the N=2 BI action implies that the complex scalar $|W| = a = P + iQ$ is the Goldstone field associated with two spontaneously broken translations (in the directions orthogonal to a D3-brane worldvolume in six dimensions). Hence, the action (10) or (12) should possess hidden invariance with respect to spontaneously broken (non-linearly realized) translations, $\delta a = \lambda + \dots$, where λ is the complex (rigid) parameter. This symmetry is obvious from the viewpoint of a (1,0) supersymmetric BI action in six dimensions [1], which is related to the four-dimensional N=2 BI action via dimensional reduction. Indeed, the six-dimensional action depends upon its gauge fields via their field strength only, while one can identify $A_4 + iA_5 = a$. Hence, the dimensionally reduced action actually depends upon the derivatives of a , and not upon a itself, though it is not manifest in eq. (10). Similarly, the spinor components ψ_α^i of W in eq. (8) are supposed to be the Goldstone fermions associated with two spontaneously broken (non-linearly realized) supersymmetries in four dimensions, $\delta\psi_\alpha^i = \lambda_\alpha^i + \dots$, where λ_α^i are the (rigid) spinor parameters. To the best of our knowledge, explicit transformation laws of the spontaneously broken symmetries were not found earlier.

Spontaneously broken symmetries unambiguously fix the corresponding Goldstone action. Since, in our case, the N=2 BI action is unambiguously determined by the non-linear constraint (13), there should be a simple relation between the non-linear transformations in question and the constraint (8). Once their relation is appreciated, it is not difficult to find the corresponding transformation laws in N=2 superspace,

$$\delta X = 2\Lambda W, \quad \delta W = \Lambda \left(1 - \frac{1}{4}\bar{D}^4 \bar{X}\right) - \frac{X}{\bar{W}} \bar{D}^4 (\bar{W} \bar{\Lambda}), \quad (15)$$

where Λ is the spacetime-independent (rigid) N=2 superfield parameter,

$$\Lambda = \lambda + \theta_i^\alpha \lambda_\alpha^i + \frac{i}{8} \theta_i^\alpha (\sigma^{\mu\nu})_\alpha{}^\beta \theta_\beta^i \lambda_{\mu\nu}, \quad (16)$$

and X is the solution to the non-linear constraint (13). The second equation (15) thus contains terms of all orders in W and \bar{W} . The invariance of the action (12) under the transformations (15) is the simple consequence of the fact that

$$D^4 W, \quad (D^3)_i^\alpha W \quad \text{and} \quad D_{\alpha\beta} W \quad (17)$$

are total derivatives in x -space, because of eqs. (8) and (9). The second relation (15) follows from the first one by varying the constraint (13). Comparing eqs. (8) and (15) shows that λ is the rigid parameter of broken translations, whereas λ_α^i are the rigid parameters of two broken supersymmetries. Surprisingly enough, there exists yet another non-linear symmetry with the rigid parameters $\lambda_{\mu\nu}$, which is apparently related to the Goldstone nature of the Maxwell field itself.

It is possible to rewrite the action (12) into the ‘free-field’ form (subject to the non-linear constraint)

$$S = \frac{1}{2} \int d^4x d^4\theta W^2 + \frac{1}{8} \int d^4x d^8\theta \bar{X} X , \quad (18)$$

where I have merely substituted the constraint (13) into eq. (12) and used eq. (6). Equation (18) is the N=2 analogue to the known ‘free-field’ form of the N=1 BI action, given by a sum of free actions for an N=1 vector multiplet and an N=1 chiral multiplet, related by a non-linear constraint [3, 2]. There is, however, the obvious difference between the N=1 and N=2 ‘free-field’ actions, because the second term in eq. (18) gives rise to higher derivatives in components. The existence of the field redefinition eliminating the higher derivatives is guaranteed by the existence of the equivalent (gauge-fixed) D3-brane action without higher derivatives but with non-manifest (non-linearly realized) unbroken N=2 supersymmetry [1, 2].

4 Outlook

The N=2 BI theory can be made (rigidly) N=2 superconformally invariant by modifying the constraint (13) as (*cf.* ref. [5])

$$X = \frac{1}{4} \frac{X}{\Phi^2} \bar{D}^4 \left(\frac{\bar{X}}{\bar{\Phi}^2} \right) + W^2 , \quad (19)$$

where the N=2 ‘superconformal compensator’ Φ is an N=2 restricted chiral superfield obeying the constraints (7). The original N=2 BI action is obtained from eqs. (12) and (19) by ‘freezing’ Φ to $\Phi = 1/\sqrt{b} = (2\pi\alpha')^{-1/2}$.

The bosonic BI lagrangian (sect. 2) interpolates between the Maxwell lagrangian, $-\frac{1}{4}F^{\mu\nu}F_{\mu\nu} \equiv -\frac{1}{4}F^2$, for small F and the total derivative, $\frac{i}{8}\varepsilon^{\mu\nu\lambda\rho}F_{\mu\nu}F_{\lambda\rho} \equiv \frac{i}{4}F\tilde{F}$, for large F [2]. In the $b \rightarrow 0$ limit the BI lagrangian reduces to

$$\frac{F^2}{F\tilde{F}} , \quad (20)$$

whose N=2 supersymmetric extension [6]

$$\int d^8\theta \frac{W^2 \bar{W}^2}{K \bar{K}} \left(\frac{K + \bar{K}}{K - \bar{K}} \right) \quad (21)$$

follows from eq. (11) in the $b \rightarrow 0$ limit.

One may, therefore, think of Φ as a *constant* non-covariant background containing a constant antisymmetric tensor $B_{\mu\nu}$ on the place of $F_{\mu\nu}$ in eq. (8). The $b \rightarrow 0$ limit is

then described by sending Φ to infinity, at large $B_{\mu\nu}$ in particular. The N=2 BI action in this limit is believed to be equivalent to a rank-one (Maxwell) *noncommutative* N=2 supersymmetric gauge field theory via Seiberg-Witten map [7], with $B_{\mu\nu}$ being the measure of noncommutativity in x -space,

$$[x^\mu, x^\nu] = i(B^{-1})^{\mu\nu} . \quad (22)$$

Acknowledgement

It is my pleasure to thank Professor S. J. Gates Jr. for kind hospitality extended to me at the University of Maryland in College Park during preparation of this paper.

References

- [1] S. V. Ketov, Mod. Phys. Lett. **A14** (1999) 501; Nucl. Phys. **B553** (1999) 250
- [2] A. A. Tseytlin, *Born-Infeld action, supersymmetry, and string theory*, hep-th/9908105
- [3] J. Bagger and A. Galperin, Phys. Rev. **D55** (1997) 1091;
M. Roček and A. A. Tseytlin, Phys. Rev. **D59** (1999) 106001
- [4] S. V. Ketov, *Quantum Non-Linear Sigma-Models*, Springer-Verlag, 2000, Ch. IV
- [5] S. M. Kuzenko and S. Theisen, JHEP 0003:034 (2000)
- [6] S. Ferrara and M. A. Lledo, *Some aspects of deformations of supersymmetric field theories*, hep-th/0002084
- [7] N. Seiberg and E. Witten, JHEP 9909:032 (1999).